NUMBER THEORY.

215. Proposed by R. D. CARMICHAEL, Indiana University.

Find one or more values of n such that a polygon of n sides shall have the number of its diagonals equal to the cube of an integer.

SOLUTION BY WALTER C. EELLS, U. S. Naval Academy.

Since the number of diagonals of an n-sided polygon is $\frac{n(n-3)}{2}$, this is equivalent to the problem: find solutions in integers of $n^2 - 3n = 2k^3$. By the aid of a table of squares it is easily found that the only values of n less than 1,000 satisfying this equation are 9 and 128, for which $k = 3$ and 20 respectively, the number of diagonals being 27 and 8,000.

216. Proposed by ELIJAH SWIFT, University of Vermont.

If p is a prime > 3, show that $\Sigma_{a=1}^{a=p-1} 1/a \equiv 0 \pmod{p^2}$, where $1/a \equiv x$, if $ax \equiv 1 \pmod{p^2}$.

SOLUTION BY THE PROPOSER.

Referring to my solution[1] of algebra problem number 385, I proved that $A_{p-2} \equiv 0 \pmod{p^2}$, where $A_{p-2} \equiv 1\cdot 2\cdot 3 \cdots p-2 + 1\cdot 3\cdot 4 \cdots p-1 + \cdots$. Hence,

$$A_{p-2} \equiv (1\cdot 2\cdot 3 \cdots (p-1)) \sum_{a=1}^{p-1} \frac{1}{a}.$$

Suppose that

$$1\cdot 2\cdot 3 \cdots (p-1) \equiv -1 + A\cdot p \pmod{p^2}.$$

Then

$$A_{p-2} \equiv \sum_{a=1}^{p-1} \frac{-1 + Ap}{a} = \sum_{a=1}^{p-1} \frac{-1}{a} + pA \sum_{a=1}^{p-1} \frac{1}{a}.$$

But

$$\sum_{a=1}^{a=p-1} \frac{1}{a} \equiv \sum_{a=1}^{a=p-1} a \pmod{p} \equiv 0,$$

since

$$A_{p-2} \equiv 0 \pmod{p^2} \sum_{a=1}^{a=p-1} \frac{1}{a} \equiv 0 \pmod{p^2}.$$

NOTES AND NEWS.

EDITED BY W. DEW. CAIRNS.

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Mr. FORREST R. BAKER, assistant in mathematics at the University of Michigan, died December 6, following an operation for appendicitis.

[1] Volume XXI, page 157, May, 1914.